Nomenclature, Symbols and Classification of the Subperiodic Groups

Report of a Working Group of the International Union of Crystallography Commission on Crystallographic Nomenclature

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Introduction

The subperiodic groups consist of 1) The 7 types of *frieze groups*, two-dimensional groups with one-dimensional translations, 2) The 75 types of *rod groups*, three-dimensional groups with one-dimensional translations, and 3) The 80 types of *layer groups*, three-dimensional groups with two-dimensional translations. A survey of the nomenclature used for these three varieties of groups is given in Appendix A.

Volume E of the International Tables For Crystallography is, in part, an extension of the *International Tables For Crystallography, Volume A: Space-Group Symmetry (ITC(1987))*. Symmetry tables are given in *ITC(1987)* for the 230 types of three-dimensional space groups and the 17 types of two-dimensional space groups. We give in *Volume E* analogous symmetry tables for the subperiodic groups.

The symmetry tables of the subperiodic groups are but one of three parts of *Volume E* (Kopsky and Litvin (1990)). The second part considers the relationships among subperiodic groups, and between subperiodic groups and space groups, i.e. between the three-dimensional layer groups and rod groups and the three-dimensional space groups, and between the two-dimensional frieze groups and the two-dimensional space groups. The third part considers the symmetries of planes in crystals, which involves again relationships between the three-dimensional layer groups and the three-dimensional space groups.

The subperiodic groups are not considered independently of the space groups. The two-dimensional frieze groups are considered in the context of a two-dimensional space along with their concomitant relationships with two-dimensional space groups. The three-dimensional rod and layer groups are considered in the context of a three-dimensional space along with their concomitant relationships with three-dimensional space groups. These relationships are the basis for the symbols and classification of the subperiodic groups which are used in *Volume E*. (See also introduction to Appendix B.)

Conventional coordinate systems

Subperiodic groups are described, as are the space groups, see *ITC(1987)*, by means of a *crystallographic coordinate system*, consisting of a *crystallographic basis* and a *crystallographic origin*. For subperiodic groups, not all basis vectors of the crystallographic basis are lattice vectors. For the three-dimensional layer groups and rod groups, the basis vectors are labeled ***a***, ***b***, and ***c***. The basis vectors ***a*** and ***b*** are chosen as lattice vectors in the case of layer groups, and ***c*** is chosen as a lattice vector in the case of rod groups. For the two-dimensional frieze groups, the basis vectors are denoted by ***a*** and ***b*** with ***a*** chosen as a lattice vector.

The selection of a crystallographic coordinate system is not unique. Following *ITC(1987)*, we conventionally choose a right-handed set of basis vectors such that the symmetry of the subperiodic group is best displayed. Restrictions on the conventional coordinate system due to the symmetry of the subperiodic groups are listed in the sixth column of Table 5 for the layer groups, and the fourth column of Tables 6 and 7 for the rod and frieze groups, respectively. The *crystallographic origin* is conventionally chosen at a center of symmetry or at a point of high site symmetry. The *conventional unit cell* of a subperiodic group is defined by the conventional crystallographic origin and by the basis vectors of the conventional crystallographic coordinate system which are also lattice vectors. For layer groups, the cell parameters associated with the lattice vectors to be determined to specify the conventional unit cell are given in the seventh column of Table 5. The conventional unit cell obtained in this manner turns out to be either *primitive* or *centered* as denoted by *p* or *c* in the eighth column of Table 5. For rod and frieze groups, the single cell parameter to be specified is the length of the basis vector which is also a lattice vector.

International (Hermann-Mauguin) symbols for subperiodic groups

Both the short and the full Hermann-Mauguin symbols consist of two parts, (i) a letter indicating the centering type of the conventional cell, and (ii) a set of characters indicating symmetry elements of the subperiodic group.

(i) The letters for the two centering types for layer groups are the lower case letters p for a primitive cell and c for a centered cell. For rod and frieze groups, there is only one centering type, the one-dimensional primitive cell which is denoted by the lower case script *p* (*ITC(1987)*).

(ii) The one or three entries after the centering letter refer to the one or three kinds of *symmetry directions* of the conventional crystallographic basis. Symmetry directions occur either as singular directions or as sets of symmetrically equivalent symmetry directions. Only one representative of each set is required. The sets of symmetry directions and their sequence in the Hermann-Mauguin symbol are summarized in Table 1. In the first column we give a classification of the subperiodic groups explained below. In the remaining columns we give the symmetry directions and their sequence. Directions which belong to the same set of equivalent symmetry directions are enclosed in a single box. The top entry in each set is taken as the representative of that set. Note that these symmetry directions and sequences are identical for two-dimensional subperiodic groups and two-dimensional space groups, and for three-dimensional subperiodic groups and three-dimensional space groups. (See Table 2.4.1 of *ITC(1987)*.)

Each position in the Hermann-Mauguin subperiodic group type symbol contains one or two characters designating symmetry elements, axes and planes, of the subperiodic group that occur for the corresponding conventional crystallographic basis symmetry direction. Symmetry planes are represented by their normals; if asymmetry axis and a normal to a symmetry plane are parallel, the two characters are separated by a slash, e.g. the 4/m in *p*4/mcc
(R40). (The letters L, R, and F denote layer, rod, and frieze groups, respectively. The number following the letter is the subperiodic group type's sequential numbering in the listings given in Tables 2, 3, and 4.) Conventional crystallographic basis symmetry directions which carry no symmetry elements for the subperiodic group are denoted by the symbol "1", e.g. p3m1 (L69) and p112 (L2). If no misinterpretation is possible,

entries "1" at the end of a subperiodic group symbol are omitted, as p4 (L49) instead of p411. Subperiodic group types which have in addition to translations no symmetry directions or only centers of symmetry have only one entry after the centering letter. These are the layer group types p1 (L1) and $p\bar{1}$ (L2), the rod group types *p*1 (R1) and $p\bar{1}$ (R2), and the frieze group *p*1 (F1).

The listings of the frieze, rod, and layer group type symbols are given, respectively, in Tables 2, 3, and 4. In the first column is the sequential numbering and in the second column is the short Hermann-Mauguin symbol. In the third column is the full Hermann-Mauguin symbol if distinct from the short symbol. For the two layer groups L5 and L7, the three symbols for the three cell choices are given between square brackets.

The resulting Hermann-Mauguin symbols for the subperiodic group types are distinct except for the rod and frieze group types *p*1 (R1, F1), *p*211 (R3, F2), and *p*11m (R10, F4). The resulting Hermann-Mauguin symbols for the subperiodic group types are distinct from those of the space group types except for the layer and two-dimensional space group types p1 (L1, 2DSG1), p3 (L65, 2DSG13), p3m1 (L69, 2DSG14), p31m (L70, 2DSG15), p6 (L73, 2DSG16), and p6mm (L77, 2DSG17). (In Appendix B we give a survey of the sets of symbols which have been introduced for subperiodic groups.)

Classification of Subperiodic Groups

In analogy with the *(crystallographic) space-group types,* the subperiodic groups are classified into *(crystallographic) subperiodic-group types*: There are 80 (crystallographic) layer group types, 75 (crystallographic) rod group types, and 7 (crystallographic) frieze group types. This is the classification used in the tabulations of the subperiodic groups given in Sections 3, 4, and 5. In analogy with the *affine space-group types*, the subperiodic groups can be classified in *affine subperiodic-group types:* There are 80 affine layer group types, 67 affine rod group types, and 7 affine frieze group types. For layer and frieze groups, the classification of subperiodic groups into affine subperiodic group types is identical with the classification into (crystallographic) subperiodic group types. In the case of rod groups, there are eight affine rod group types that split into pairs of *enantiomorphic rod group types*. The eight pairs of enantiomorphic rod group types are $p4_1$(R24) - $p4_3$(R26), $p4_122$(R31) - $p4_322$(R33), $p3_1$(R43) - $p3_2$(R44), $p3_112$(R47) - $p3_212$(R48), $p6_1$(R54) - $p6_5$(R58), $p6_2$(R55) - $p6_4$(R57), $p6_122$(R63) - $p6_522$(R67), and $p6_222$(R64) - $p6_422$(R66).

The classification of subperiodic groups according to (geometric) crystal classes is according to the crystallographic point group to which the subperiodic group belongs. There are 27 (geometric) crystal classes of layer groups and rod groups, and 4 (geometric) crystal classes of frieze groups. These are listed, for layer groups, in the fourth column of Table 5, and for the rod and frieze groups in the second column of Tables 6 and 7 respectively.

We classify all subperiodic groups according the following classifications of the subperiodic group's point group and lattice group. These classifications emphasize that we are considering subperiodic groups, i.e groups with translations which span a lower dimensional space. The layer and rod groups are three-dimensional groups with, respectively, two- and one-dimensional translations, and the frieze groups are two-dimensional groups with one-dimensional translations. These classifications also emphasize the relationships between subperiodic groups and space groups:

The point group of each layer and rod group is a three-dimensional point group, and the point group of each frieze group is a two-dimensional point group. The classification into *crystal systems*, see *ITC(1987)*, is a classification of crystallographic point group types. Three-dimensional point groups are classified into the triclinic, monoclinic, orthorhombic, tetragonal, trigonal, hexagonal, and cubic crystal systems. Two-dimensional point groups are classified into oblique, rectangular, square, and hexagonal crystal systems. We shall use this crystal system classification scheme to classify the subperiodic groups according to their point groups. Consequently, the three-dimensional subperiodic groups are classified, see the third column of Table 5 and the first column of Table 6, into the triclinic, monoclinic, orthorhombic, tetragonal, trigonal, and hexagonal crystal systems. The cubic crystal system does not arise for three-dimensional subperiodic groups. The two-dimensional subperiodic groups are classified, see the first column of Table 7, into oblique and rectangular crystal systems. The square and hexagonal crystal systems do not arise for two-dimensional subperiodic groups.

The lattice of each layer group is a two-dimensional lattice, and the lattice of each rod and frieze group is a one-dimensional lattice. The classification into *Bravais (flock) systems*, see *ITC(1987)*, is a classification of lattice types. Two-dimensional lattice types are classified into the oblique, rectangular, square, and hexagonal Bravais systems. One-dimensional lattice types are classified into a single Bravais system. We shall use this Bravais system classification scheme to classify the subperiodic groups according to their lattices. Consequently, layer groups are classified , see column one of Table 5, into oblique, rectangular, square, and hexagonal Bravais systems. For rod and frieze groups, which have one-dimensional lattices, no classification is explicitly given in Tables 6 and 7 as all one-dimensional lattices are classified into a single Bravais system. A subdivision of the monoclinic rod group crystal system is made, see column one of Table 6, into two subdivisions: monoclinic/inclined and monoclinic/orthogonal. This subdivision is based on which of two conventional coordinate systems is used for the rod groups of the monoclinic crystal system, and the orientation of the plane containing the non-lattice basis vectors relative to the lattice vectors. For the monoclinic/inclined subdivision, $\beta = \gamma = 90^{\circ}$, see column four of Table 6, and the plane containing the ***a*** and ***b*** non-lattice basis vectors is, see Figure 1, *inclined* with respect to the lattice basis vector ***c***. For the monoclinic/orthogonal subdivision, $\alpha = \beta = 90^{\circ}$ and the plane containing the ***a*** and ***b*** non-lattice basis vectors is, see Figure 2, *orthogonal* to the lattice basis vector ***c***.

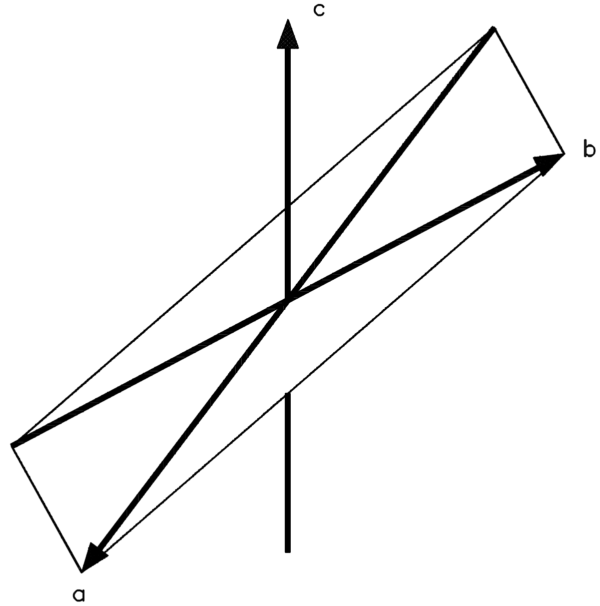

Figure 1: For the monoclinic/inclined subdivision, $ = (= 90° and the plane containing the ***a*** and ***b*** non-lattice basis vectors is *inclined* with respect to the lattice basis vector ***c***.

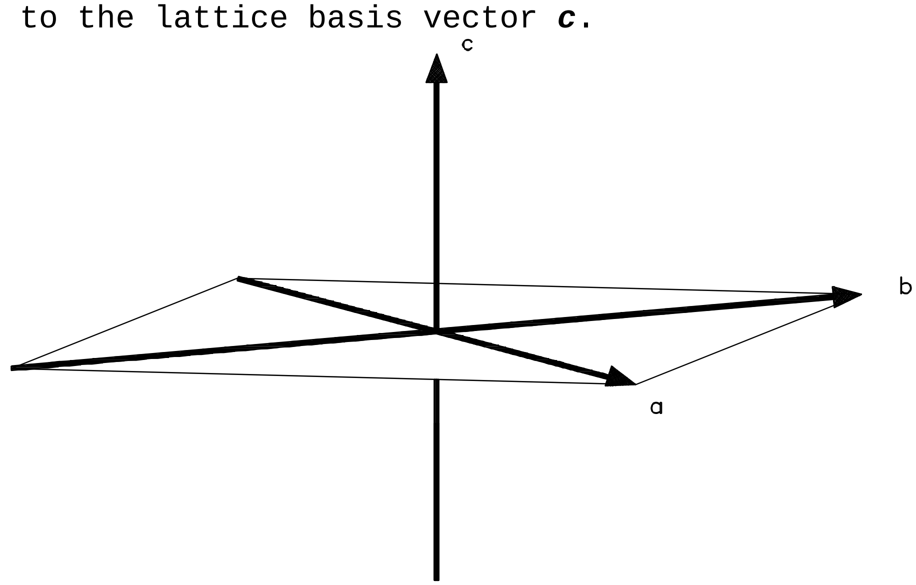

Figure 2: For the monoclinic/orthogonal subdivision, " = $ = 90° and the plane containing the ***a*** and ***b*** non-lattice basis vectors is *orthogonal* to the lattice basis vector ***c***.

Table 1: Sets of symmetry directions[(*)] and their position in the Hermann-Mauguin symbol

	Symmetry direction (position in Hermann-Mauguin symbol)		
	Primary	Secondary	Tertiary
Layer groups and rod groups			
Triclinic	None		
Monoclinic Orthorhombic	[1 0 0]	[0 1 0]	[0 0 1]
Tetragonal	[0 0 1]	[1 0 0] [0 1 0]	[1 $\bar{1}$ 0] [1 1 0]
Trigonal Hexagonal	[0 0 1]	[1 0 0] [0 1 0] [$\bar{1}$ $\bar{1}$ 0]	[1 $\bar{1}$ 0] [1 2 0] [$\bar{2}$ $\bar{1}$ 0]
Frieze groups			
Oblique	Rotation point in plane		
Rectangular		[1 0]	[0 1]

[(*)] Periodic directions are [1 0 0] and [0 1 0] for the Layer Groups, [0 0 1] for the Rod Groups, and [1 0] for the Frieze Groups.

Table 2: Frieze Group Symbols

Oblique	
F1	*p*1
F2	*p*211
Rectangular	
F3	*p*1m1
F4	*p*11m
F5	*p*11g
F6	*p*2mm
F7	*p*2mg

Table 3: Rod Groups Symbols

Triclinic

R1	$p1$	
R2	$p\bar{1}$	

Monoclinic/Inclined

R3	$p211$	
R4	$pm11$	
R5	$pc11$	
R6	$p2/m11$	
R7	$p2/c11$	

Monoclinic/Orthogonal

R8	$p112$	
R9	$p112_1$	
R10	$p11m$	
R11	$p112/m$	
R12	$p112_1/m$	

Orthorhombic

R13	$p222$	
R14	$p222_1$	
R15	$pmm2$	
R16	$pcc2$	
R17	$pmc2_1$	
R18	$p2mm$	
R19	$p2cm$	
R20	$pmmm$	$p2/m2/m2/m$
R21	$pccm$	$p2/c2/c2/m$

R22	$pmcm$	$p2/m2/c2_1/m$

Tetragonal

R23	$p4$	
R24	$p4_1$	
R25	$p4_2$	
R26	$p4_3$	
R27	$p\bar{4}$	
R28	$p4/m$	
R29	$p4_2/m$	
R30	$p422$	
R31	$p4_122$	
R32	$p4_222$	
R33	$p4_322$	
R34	$p4mm$	
R35	$p4_2cm$	
R36	$p4cc$	
R37	$p\bar{4}2m$	
R38	$p\bar{4}2c$	
R39	$p4/mmm$	$p4/m2/m2/m$
R40	$p4/mcc$	$p4/m2/c2/c$
R41	$p4_2/mmc$	$p4_2/m2/m2/c$

Trigonal

R42	$p3$	
R43	$p3_1$	
R44	$p3_2$	
R45	$p\bar{3}$	
R46	$p312$	

R47	$p3_112$
R48	$p3_212$
R49	$p3m1$
R50	$p3c1$
R51	$p\bar{3}1m$
R52	$p\bar{3}1c$

Hexagonal

R53	$p6$
R54	$p6_1$
R55	$p6_2$
R56	$p6_3$
R57	$p6_4$
R58	$p6_5$
R59	$p\bar{6}$
R60	$p6/m$
R61	$p6_3/m$
R62	$p622$
R63	$p6_122$
R64	$p6_222$
R65	$p6_322$
R66	$p6_422$
R67	$p6_522$
R68	$p6mm$
R69	$p6cc$
R70	$p6_3mc$
R71	$p\bar{6}m2$

R72	$p\bar{6}c2$	
R73	$p6/mmm$	$p6/m2/m2/m$
R74	$p6/mcc$	$p6/m2/c2/c$
R75	$p6_3/mmc$	$p6_3/m2/m2/c$

Table 4: Layer Group Symbols

Triclinic/Oblique

L1	p1	
L2	$p\bar{1}$	

Monoclinic/Oblique

L3	p112	
L4	p11m	
L5	p11a	[p11a; p11n; p11b]
L6	p112/m	
L7	p112/a	[p112/a; p112/n; p112/b]

Monoclinic/Rectangular

L8	p211	
L9	$p2_111$	
L10	c211	
L11	pm11	
L12	pb11	
L13	cm11	
L14	p2/m11	
L15	$p2_1/m11$	
L16	p2/b11	
L17	$p2_1/b11$	
L18	c2/m11	

Orthorhombic/Rectangular

L19	p222	
L20	$p2_122$	

L21	$p2_12_12$	
L22	c222	
L23	pmm2	
L24	pma2	
L25	pba2	
L26	cmm2	
L27	pm2m	
L28	$pm2_1b$	
L29	$pb2_1m$	
L30	pb2b	
L31	pm2a	
L32	$pm2_1n$	
L33	$pb2_1a$	
L34	pb2n	
L35	cm2m	
L36	cm2a	
L37	pmmm	p2/m2/m2/m
L38	pmaa	p2/m2/a2/a
L39	pban	p2/b2/a2/n
L40	pmam	$p2_1/m2/a2/m$
L41	pmma	$p2_1/m2/m2/a$
L42	pman	$p2/m2_1/a2/n$
L43	pbaa	$p2/b2_1/a2/a$
L44	pbam	$p2_1b2_1/a2/m$
L45	pbma	$p2_1/b2_1/m2/a$
L46	pmmn	$p2_1/m2_1/m2/n$

L47	cmmm	c2/m2/m2/m
L48	cmma	c2/m2/m2/a

<u>Tetragonal/Square</u>

L49	p4	
L50	$p\bar{4}$	
L51	p4/m	
L52	p4/n	
L53	p422	
L54	$p42_12$	
L55	p4mm	
L56	p4bm	
L57	$p\bar{4}2m$	
L58	$p\bar{4}2_1m$	
L59	$p\bar{4}m2$	
L60	$p\bar{4}b2$	
L61	p4/mmm	p4/m2/m2/m
L62	p4/nbm	p4/n2/b2/m
L63	p4/mbm	$p4/m2_1/b2/m$
L64	p4/nmm	$p4/n2_1/m2/m$

<u>Trigonal/Hexagonal</u>

L65	p3
L66	$p\bar{3}$
L67	p312
L68	p321
L69	p3m1
L70	p31m

L71	p$\bar{3}$1m	
L72	p$\bar{3}$m1	

Hexagonal/Hexagonal

L73	p6	
L74	p$\bar{6}$	
L75	p6/m	
L76	p622	
L77	p6mm	
L78	p$\bar{6}$m2	
L79	p$\bar{6}$2m	
L80	p6/mmm	p6/m2/m2/m

Table 5: Classification of layer groups

Two-dimensional lattice Bravais system	Symbol	Three-dimensional point group crystal system	Crystallographic point groups#	No. of groups	Conventional coordinate system restrictions	Cell parameters to be determined	Bravais lattice
LAYER GROUPS							
Oblique	*m*	Triclinic	1, :$\bar{1}$:	2	None	*a*,*b*,(##	mp
		Monoclinic	2, m, *2/m*	5	" = $ = 90º		
Rectangular	*o*	Monoclinic		11	$ = (= 90º	*a*,*b*	op oc
		Orthorhombic	222, 2mm, *mmm*	30	" = $ = (= 90º		
Square	*t*	Tetragonal	4, $\bar{4}$, :4/m:, 422, 4mm, $\bar{4}$2m, *4/mmm*	16	" = $ = (= 90º *a* = *b*	*a*	tp
Hexagonal	*h*	Trigonal	3, :$\bar{3}$:, 32, 3m, :$\bar{3}$m:	8	" = $ = 90º	*a*	hp
		Hexagonal	6, $\bar{6}$, :6/m: 622, 6mm, $\bar{6}$m2, *6/mmm*	8	(= 120º *a* = *b*		

\# Symbols surrounded by dashed or full lines indicate Laue classes. Full lines indicate point groups which are also lattice point symmetries (holohedries).
\## This angle is taken conventionally to be non-acute, i.e. ≥ 90º.

Table 6: Classification of rod groups

Three-dimensional point group crystal system	Crystallographic point groups#	No. of groups	Conventional coordinate system restrictions
ROD GROUPS			
Triclinic	1, $\boxed{\bar{1}}$	2	None
Monoclinic (Inclined)	2, m, $\boxed{2/m}$	5	$\beta = \gamma = 90^{\circ}$
Monoclinic (Orthogonal)	2, m, $\boxed{2/m}$	5	$\alpha = \beta = 90^{\circ}$
Orthorhombic	222, 2mm, $\boxed{mmm}$	10	$\alpha = \beta = \gamma = 90^{\circ}$
Tetragonal	4, $\bar{4}$, $\boxed{4/m}$, 422, 4mm, $\bar{4}2m$, $\boxed{4/mmm}$	19	$\alpha = \beta = \gamma = 90^{\circ}$
Trigonal	3, $\boxed{\bar{3}}$, 32, 3m, $\boxed{\bar{3}m}$	11	$\alpha = \beta = 90^{\circ}$ $\gamma = 120^{\circ}$
Hexagonal	6, $\bar{6}$, $\boxed{6/m}$, 622, 6mm, $\bar{6}m2$, $\boxed{6/mmm}$	23	$\alpha = \beta = 90^{\circ}$ $\gamma = 120^{\circ}$

Boxed symbols indicate Laue classes.

Table 7: Classification of frieze groups

Two-dimensional point group crystal system	Crystallographic point groups#	No. of groups	Conventional coordinate system restrictions
FRIEZE GROUPS			
Oblique	1, *2*	2	None
Rectangular	m, *2mm*	5	(= 90^o

Boxed symbols indicate Laue classes.

Appendix A: Subperiodic Group Nomenclature

There exists a wide variety of nomenclature for subperiodic groups. The frieze groups (Holser (1961), Bohm & Dornberger-Schiff (1966, 1967)) have been named *line groups (borders) in two-dimensions* (IT (1952)), groups of *bortenornamente* (Speiser (1956)), *bandgruppen* (Niggli (1924)), *ribbon groups* (Kohler (1977)), groups of *one-sided bands* (Shubnikov & Koptsik (1974)), *line groups in a plane* (Belov (1966)), and groups of *borders* (Vainshtein (1981)).

The rod groups have been named *line groups in three-dimensions* (IT (1952), Opechowski (1986)), *kettengruppen* (Hermann (1929a,b)), *eindimensionalen raumgruppen* (Alexander (1929, 1934)), *linear space groups* (Bohm & Dornberger-Schiff (1966, 1967)), *rod groups* (Shubnikov & Koptsik (1974), Kohler (1977), Vujicic, Bozovic, & Herbut (1977), Koch & Fisher (1978a,b,c)), and *one-dimensional (subperiodic) groups in three-dimensions* (Brown, Bulow, Neubuser, Wondratschek, & Zassenhaus (1978)).

The existence of the layer groups was recognized by several authors in the late nineteen twenties (Speiser (1956), Hermann (1929a,b), Alexander & Herrmann (1928, 1929a,b), Weber (1929), Heesch (1929)). These and subsequent authors (Cochran (1952), Dornberger-Schiff (1956), Belov & Tarkhova (1956a,b), Belov (1959), Wood (1964a,b), Chapuis (1966), Bohn & Dornberger-Schiff (1966,1967), Shubnikov & Koptsik (1974), Kohler (1977), Goodman (1984), Grell, Krause, & Grell (1989), Litvin (1989)) have introduced a wide variety of nomenclature for these groups. These groups have been named *net groups* (IT (1952), Opechowski (1986)), *netzgruppen* (Hermann (1929a,b)), *zweidimensionale Raumgruppen*

(Alexander & Herrmann (1928, 1929a,b)), *diperiodic groups in three-dimensions* (Wood (1964)), *black and white space groups in two-dimensions* (Mackay (1957)), *layer space groups* (Shubnikov & Koptsik (1974)), *layer groups* (Kohler (1977), Vainshtein (1981), Koch & Fisher (1978a,b,c)), *two-dimensional (subperiodic) groups in three-dimensional space* (Herman (1928)), and *plane space groups in three-dimensions* (Grell, Krause, & Grell (1989)).

Appendix B: Subperiodic Group Symbols

The following general criterion has been used to select the sets of symbols in Tables 2, 3, and 4: *consistency with the symbols used for the space groups given in ITC(1987)*. Specific criteria following from this general criterion are as follows:

1) *The symbols of subperiodic groups are to be of the Hermann-Mauguin (International) type*. This is the type of symbol used for space groups in *ITC(1987)*.

2) *A symbol of a subperiodic group is to consist of a letter indicating the lattice centering type followed by a set of characters indicating symmetry elements*. This is the format of the Hermann-Mauguin (International) space group symbols in *ITC(1987)*.

3) *The sets of symmetry directions and their sequences in the symbols of the subperiodic groups are those of the corresponding space groups*. Layer and rod groups are three-dimensional subperiodic groups of the three-dimensional space groups, and frieze groups are two-dimensional subperiodic groups of the two-dimensional space groups. Consequently, the symmetry directions and sequence of the characters indicating symmetry elements in layer and rod groups are those of the three-dimensional space groups; in frieze groups, they are those of the two-dimensional space groups, see Table 2.4.1 of *ITC(1987)* and Table 1 above.

A consequence of this criterion is a lucidity of notation in the sections of Volume E which deal with the relationships between space groups and their subperiodic groups. In these sections, e.g. layer groups appear as subgroups of the three-dimensional space groups, as factor groups of the three-dimensional space groups, and as the symmetry of planes which transect a crystal of a given three-dimensional space group symmetry. For example, the layer group pmm2 is a subgroup of the three-dimensional space group Pmm2; the layer group pmm2 is isomorphic to the factor group Pmm2/T_z of the three-dimensional space group Pmm2, where T_z is the translational subgroup of all translations along the z-axis; and the layer group pmm2 is the symmetry of the plane, transecting a crystal of three-dimensional space group symmetry Pmm2, perpendicular to the z-axis, at z=0. In these examples, this criterion leads to the easily comprehendible notation in which the symbols for the three-dimensional space group and its related subperiodic layer group differ only in the letter indicating the lattice type.

Following is a survey of the sets of symbols which have been used for the subperiodic groups. Considering these sets of symbols vis-a-vis the above criteria, leads to the sets of symbols for subperiodic groups listed in Tables 2, 3, and 4.

Layer Groups:

A list of sets of symbols for the layer groups is given in Fold-out 1. The information provided in the columns of Fold-out 1 is as follows:

Columns 1 and 2: Sequential numbering and symbols used in Table 4 and *Volume E*.

Columns 3 and 4: Sequential numbering and symbols listed by E. Wood (1964a,b) and Litvin & Wike (1991).

Columns 5 and 6: Sequential numbering and symbols listed by Bohm and Dornberger-Schiff (1966,1967).

Columns 7 and 8: Sequential numbering and symbols listed by Shubnikov and Koptsik (1974) and Vainshtein (1981).

Column 9: Symbols listed by Holser (1958).

Column 10: Sequential numbering listed by Weber (1929).

Column 11: Symbols listed by C. Hermann (1929a,b).

Column 12: Symbols listed by Alexander and K. Herrmann (1928, 1929a,b).

Column 13: Symbols listed by Niggli (Wood (1964a,b)).

Column 14: Symbols listed by Shubnikov and Koptsik (1974).

Columns 15 and 16: Symbols listed by Aroyo and Wondratschek (1987).

Column 17: Symbols listed by Belov, Neronova, and Smirnova (1957).

Columns 18 and 19: Symbols and sequential numbering listed by Belov and Neronova (1956).

Columns 20 and 21: Symbols listed by Cochran as listed, respectively, by Cochran (1952) and Belov and Neronova (1956).

Column 22: Symbols listed by Opechowski (1986).

Column 23: Symbols listed by Grunbaum and Shephard (1987).

Column 24: Symbols listed by Woods (1935a,b,c , 1936).

Column 25: Symbols listed by Coxeter (1985).

There is also a notation for layer groups, introduced by Janovec (1981), in which all elements in the group symbol which change the direction of the normal to the plane containing the translations are underlined, e.g. p4/m. However, we know of no listing of all layer group types in this notation.

Sets of symbols which are of a non-Hermann-Mauguin (International) type are the sets of symbols of the Schoenflies type (Columns 11 and 12), symbols of the Black and White symmetry type (Columns 16, 17, 18, 20, 21, 22, 24, and 25). Additional non-Hermann-Mauguin (International) type sets of symbols are those in Columns 14 and 23.

Sets of symbols which do not begin with a letter indicating the lattice centering type are the sets of symbols of the Niggli type (Columns 13 and 15). The order of the characters indicating symmetry elements in the sets of symbols, Columns 4 and 9, does not follow the sequence of symmetry directions used for three-dimensional space groups.

In the above section **Conventional Coordinate Systems** conventions have been introduced as to which of the basis vectors of the conventional coordinate system are lattice vectors. Consequently, no additional notation to a character indicating a symmetry element is needed to denote if the symmetry element is or is not along a lattice direction. This convention leads to a typographically simpler set of symbols than in Column 6 where the character indicating a symmetry element along a symmetry direction which is not a lattice direction is enclosed in parentheses. In addition, the set of symbols in Column 6 uses upper case letters to denote the layer group's two-dimensional lattice, where as in *ITC(1987)* upper case letters denote three-dimensional lattices.

Lastly, the symbols of the set of symbols in Column 8 are either identical with the symbols in Table 4 or, in some monoclinic and orthorhombic cases, the symbols in Column 8 are the second setting or alternative cell choice of the layer groups whose symbols are given in Table 4. These second setting and alternative cell choice symbols are included in the symmetry diagrams of the layer groups to appear in Volume E (Kopsky & Litvin (1990)).

Rod Groups:

A list of sets of symbols for the rod groups is given in Fold-out 2. The information provided in the columns of Fold-out 2 is as follows:

Columns 1 and 2: Sequential numbering and symbols used in Table 3 and *Volume E*.

Columns 3 and 4: Sequential numbering and symbols listed by Bohm and Dornberger-Schiff (1966,1967).

Columns 5, 6, and 7: Sequential numbering and two sets of symbols listed by Shubnikov and Koptsik (1974).

Column 8: Symbols listed by Opechowski (1986).

Column 9: Symbols listed by Niggli (Chapuis (1966).

Sets of symbols which are of a non-Hermann-Mauguin (International) type are the set of symbols in Column 6 and the Niggli type set of symbols in Column 9. The set of symbols in Column 8 does not use the lower case script letter *p*, as does *ITC(1987)*, to denote a one-dimensional lattice. The order of the characters indicating symmetry elements in the set of symbols in Column 7 does not follow the sequence of symmetry directions used for three-dimensional space groups. The set of symbols in Column 4 have the characters indicating symmetry elements along non-lattice directions enclosed in parentheses, and do not use a lower case script letter to denote the one-dimensional lattice.

Lastly, the symbols of the set of symbols in Column 4, without the parentheses and with the one-dimensional lattice denoted by a lower case script *p* , are identical with the symbols in Table 3, or in some cases, are the second setting of rod groups whose symbols are given in Table 3. These second setting symbols are included in the symmetry diagrams of the rod groups to appear in Volume E (Kopsky & Litvin (1990)).

Frieze Groups:

A list of sets of symbols for the frieze groups is given in Fold-out 3. The information provided in the columns of Fold-out 3 is as follows:

Columns 1 and 2: Sequential numbering and symbols used in Table 2 and *Volume E*.

Columns 3, 4, and 5: Symbols listed by Opechowski (1986).

Column 6: Symbols listed by Shubnikov and Koptsik (1974).

Column 7: Symbols listed by Vainshtein (1981).

Column 8: Symbols listed by Bohm and Dornberger-Schiff (1967).

Column 9: Symbols listed by Lockwood and Macmillan (1978).

Column 10: Symbols listed by Shubnikov and Koptsik (1974).

Sets of symbols which are of a non-Hermann-Mauguin (International) type are the set of symbols of the Black and White symmetry type (Column 3) and the sets of symbols in Columns 6 and 7. The sets of symbols in Columns 4, 5, and 10 do not follow the sequence of symmetry directions used for two-dimensional space groups. The sets of symbols in Columns 3, 4, 5, and 9 do not use a lower case script *p* to denote a one-dimensional lattice. The set of symbols in Column 8 uses parentheses and brackets to denote specific symmetry directions.

Lastly, the symbol "g" is used, in Table 1, to denote a glide line, a standard symbol for two-dimensional space groups (*ITC(1987)*). A letter identical with a basis vector symbol, e.g. "a" or "c" is not used to denote a glide line, as is done in the symbols of Columns 5, 6, 7, 8, and 10, as such a letter is a standard notation for a three-dimensional glide plane (*ITC(1987)*).

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To construct fold-outs:

Fold-out #1: The 12 pages of Layer Group Symbols are arranged as follows:

```
+))))))0))))))0))))))0)))))),
*      *      *      *      *
*   1  *   4  *   7  *  10  *
*      *      *      *      *
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/))))))3))))))3))))))3))))))1
*      *      *      *      *
*   2  *   5  *   8  *  11  *
*      *      *      *      *
*      *      *      *      *
/))))))3))))))3))))))3))))))1
*      *      *      *      *
*   3  *   6  *   9  *  12  *
*      *      *      *      *
*      *      *      *      *
.))))))2))))))2))))))2))))))-
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Fold-out #2: The 6 pages of Rod Group Symbols are arranged as follows:

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+))))))0)))))),
*      *      *
*   1  *   4  *
*      *      *
*      *      *
/))))))3))))))1
*      *      *
*   2  *   5  *
*      *      *
*      *      *
/))))))3))))))1
*      *      *
*   3  *   6  *
*      *      *
*      *      *
.))))))2))))))-
```

1. Layer Group Symbols

	1	2	3	4	5	6	7
Triclinic/ Oblique	1	p1	1	P1	1	P11(1)	1
	2	$p\bar{1}$	2	$P\bar{1}$	2	$P\bar{1}\bar{1}(\bar{1})$	3
Monoclinic/ Oblique	3	p112	3	P211	9	P11(2)	5
	4	p11m	4	Pm11	4	P11(m)	2
	5	p11a	5	Pb11	5	P11(b)	4
	6	p112/m	6	P2/m11	13	P11(2/m)	6
	7	p112/a	7	P2/b11	17	P11(2/b)	7
Monoclinic/ Rectangular	8	p211	8	P112	8	P12(1)	14
	9	$p2_111$	9	$P112_1$	10	$P12_1(1)$	15
	10	c211	10	C112	11	C12(1)	16
	11	pm11	11	P11m	3	P1m(1)	8
	12	pb11	12	P11a	5	P1a(1)	10
	13	cm11	13	C11m	7	C1m(1)	12
	14	p2/m11	14	P112/m	12	P12/m(1)	17
	15	$p2_1/m11$	15	$P112_1/m$	14	$P12_1/m(1)$	18
	16	p2/b11	17	P112/a	16	P12/a(1)	20
	17	$p2_1/b11$	18	$P112_1/a$	18	$P12_1/a(1)$	21
	18	c2/m11	16	C112/m	15	C12/m(1)	19
Orthorhombic/ Rectangular	19	p222	19	P222	33	P22(2)	37
	20	$p2_122$	20	$P222_1$	34	$P2_12(2)$	38
	21	$p2_12_12$	21	$P22_12_1$	35	$P2_12_1(2)$	39
	22	c222	22	C222	36	C22(2)	40
	23	pmm2	23	P2mm	19	Pmm(2)	22
	24	pma2	28	P2ma	24	Pma(2)	24
	25	pba2	33	P2ba	29	Pba(2)	26

26	cmm2	34	C2mm	30	Cmm(2)	28
27	pm2m	24	Pmm2	20	P2m(m)	9
28	pm2_1b	26	Pbm2_1	21	P2_1m(a)	30
29	pb2_1m	25	Pm2_1a	22	P2_1a(m)	11
30	pb2b	27	Pbb2	23	P2a(a)	31
31	pm2a	29	Pam2	25	P2m(b)	32
32	pm2_1n	32	Pnm2_1	28	P2_1m(n)	35
33	pb2_1a	30	Pab2_1	26	P2_1a(b)	33
34	pb2n	31	Pnb2	27	P2a(n)	34
35	cm2m	35	Cmm2	31	C2m(m)	13
36	cm2a	36	Cam2	32	Cm2(a)	36
37	pmmm	37	P2/m2/m2/m	37	P2/m2/m(2/m)	23
38	pmaa	38	P2/a2/m2/a	38	P2/m2/a(2/a)	41
39	pban	39	P2/n2/b2/a	39	P2/b2/a(2/n)	42
40	pmam	40	P2/m2_1/m2/a	41	P2/b2_1/m(2/m)	25
41	pmma	41	P2/a2_1/m2/m	40	P2_1/m2/m(2/a)	43
42	pman	42	P2/n2/m2_1/a	42	P2_1/b2/m(2/n)	44
43	pbaa	43	P2/a2/b2_1/a	43	P2/b2_1/a(2/a)	45
44	pbam	44	P2/m2_1/b2_1/a	44	P2_1/b2_1/a(2/m)	27
45	pbma	45	P2/a2_1/b2_1/m	45	P2_1/m2_1/a(2/b)	46
46	pmmn	46	P2/n2_1/m2_1/m	46	P2_1/m2_1/m(2/n)	47
47	cmmm	47	C2/m2/m2/m	47	C2/m2/m(2/m)	29
48	cmme	48	C2/a2/m2/m	48	C2/m2/m(2/a)	48
49	p4	49	P4	54	P(4)11	50
50	p$\bar{4}$	50	P$\bar{4}$	49	P($\bar{4}$)11	49
51	p4/m	51	P4/m	55	P(4/m)11	51
52	p4/n	52	P4/n	56	P(4/n)11	57

53	p422	53	P422	59	P(4)22	55
54	$p42_12$	54	$P42_12$	60	$P(4)2_12$	56
55	p4mm	55	P4mm	57	P(4)mm	52
56	p4bm	56	P4bm	58	P(4)bm	59
57	$p\bar{4}2m$	57	$P\bar{4}2m$	50	$P(\bar{4})2m$	54
58	$p\bar{4}2_1m$	58	$P\bar{4}2_1m$	51	$P(\bar{4})2_1m$	60
59	$p\bar{4}m2$	59	$P\bar{4}m2$	52	$P(\bar{4})m2$	61
60	$p\bar{4}b2$	60	$P\bar{4}b2$	53	$P(\bar{4})b2$	64
61	p4/mmm	61	P4/m2/m2/m	61	P(4/m)2/m2/m	53
62	p4/nbm	62	P4/n2/b2/m	62	P(4/n)2/b2/m	62
63	p4/mbm	63	$P4/m2_1/b2/m$	63	$P(4/m)2_1/b2/m$	58
64	p4/nmm	64	$P4/n2_1/m2/m$	64	$P(4/n)2_1/m2/m$	63
65	p3	65	P3	65	P(3)11	65
66	$p\bar{3}$	66	$P\bar{3}$	66	$P(\bar{3})11$	67
67	p312	67	P312	70	P(3)12	72
68	p321	68	P321	69	P(3)21	73
69	p3m1	69	P3m1	67	P(3)m1	68
70	p31m	70	P31m	68	P(3)1m	70
71	$p\bar{3}1m$	71	$P\bar{3}12/m$	72	$P(\bar{3})1m$	74
72	$p\bar{3}m1$	72	$P\bar{3}2/m1$	71	$P(\bar{3})m1$	75
73	p6	73	P6	76	P(6)11	76
74	$p\bar{6}$	74	$P\bar{6}$	73	$P(\bar{6})11$	66
75	p6/m	75	P6/m	77	P(6/m)11	77
76	p622	76	P622	79	P(6)22	80
77	p6mm	77	P6mm	78	P(6)mm	78
78	$p\bar{6}m2$	78	$P\bar{6}m2$	74	$P(\bar{6})m2$	69
79	$p\bar{6}2m$	79	$P\bar{6}2m$	75	$P(\bar{6})2m$	71
80	p6/mmm	80	P6/m2/m2/m	80	P(6/m)2/m2/m	79

8	9	10	11	12	13
p1	p1	1	$C_1\overline{p}$	C_1^1	1P1
$p\bar{1}$	$p\bar{1}$	2	$S_2\overline{p}$	C_i^1	$1P\bar{1}$
p112	p21	8	$C_2\overline{p}$	C_2^1	1P2
p11m	pm1	3	$C_{1h}\overline{p}\mu$	C_{1h}^1	mP1
p11b	pa1	4	$C_{1h}\overline{p}\alpha$	C_{1h}^2	aP1
p112/m	p2/m1	12	$C_{2h}\overline{p}\mu$	C_{2h}^1	mP2
p112/b	p2/a1	13	$C_{2h}\overline{p}\alpha$	C_{2h}^2	aP2
p121	p12	9	$D_1\overline{p}1$	C_2^2	1P12
$p12_11$	$p12_1$	10	$D_1\overline{p}2$	C_2^3	$1P12_1$
c121	c12	11	$D_1\overline{c}1$	C_2^4	1C12
p1m1	p1m	5	$C_{1v}\overline{p}\mu$	C_{1h}^3	1P1m
p1a1	p1b	6	$C_{1v}\overline{p}\beta$	C_{1h}^4	1P1g
c1m1	c1m	7	$C_{1v}\overline{c}\mu$	C_{1h}^5	1C1m
p12/m1	p12/m	14	$D_{1d}\overline{p}\mu1$	C_{2h}^3	1P12/m
$p12_1/m1$	$p12_1/m$	15	$D_{1d}\overline{p}\mu2$	C_{2h}^5	$1P12_1/m$
p12/a1	p12/b	18	$D_{1d}\overline{p}\beta2$	C_{2h}^6	1P12/g
$p12_1/a1$	$p12_1/b$	17	$D_{1d}\overline{p}\beta1$	C_{2h}^4	$1P12_1/g$
c12/m1	c12/m	16	$D_{1d}\overline{c}\mu1$	C_{2h}^7	1C12/m
p222	p222	33	$D_2\overline{p}11$	V^1	1P222
$p2_122$	$p222_1$	34	$D_2\overline{p}12$	V^3	$1P222_1$
$p2_12_12$	$p22_12_1$	35	$D_2\overline{p}22$	V^2	$1P22_12_1$
c222	c222	36	$D_2\overline{c}11$	V^4	1C222
pmm2	p2mm	19	$C_{2v}\overline{p}\mu\mu$	C_{2v}^1	1P2mm
pbm2	p2ma	20	$C_{2v}\overline{p}\mu\alpha$	C_{2v}^2	1P2mg
pba2	p2ba	21	$C_{2v}\overline{p}\beta\alpha$	C_{2v}^{10}	1P2gg

cmm2	c2mm	22	$C_{2v}\overline{c}\,\mu\mu$	C_{2v}^{3}	1C2mm
p2mm	pm2m	23	$D_{1h}\overline{p}\,\mu\mu$	C_{2v}^{4}	mP12m
$p2_1ma$	$pa2_1m$	25	$D_{1h}\overline{p}\,\beta\mu$	C_{2v}^{5}	$aP12_1m$
$p2_1am$	$pm2_1a$	24	$D_{1h}\overline{p}\,\mu\beta$	C_{2v}^{7}	$mP12_1g$
p2aa	pa2a	26	$D_{1h}\overline{p}\,\beta\beta$	C_{2v}^{6}	aP12g
p2mb	pb2m	27	$D_{1h}\overline{p}\,\alpha\mu$	C_{2v}^{11}	bP12m
$p2_1mn$	$pn2_1m$	30	$D_{1h}\overline{p}\,\upsilon\mu$	C_{2v}^{13}	$nP12_1m$
$p2_1ab$	$pb2_1a$	28	$D_{1h}\overline{p}\,\alpha\beta$	C_{2v}^{14}	$bP12_1g$
p2an	pn2a	29	$D_{1h}\overline{p}\,\upsilon\beta$	C_{2v}^{12}	nP12g
c2mm	cm2m	31	$D_{1h}\overline{c}\,\mu\mu$	C_{2v}^{8}	mC12m
c2mb	cb2m	32	$D_{1h}\overline{c}\,\alpha\mu$	C_{2v}^{9}	aC12m
pmmm	p2/m2/m2/m	37	$D_{2h}\overline{p}\,\mu\mu\mu$	V_{h}^{1}	mP2mm
pmaa	p2/a2/m2/a	38	$D_{2h}\overline{p}\,\alpha\mu\alpha$	V_{h}^{5}	aP2mg
pban	p2/n2/b2/a	39	$D_{2h}\overline{p}\,\upsilon\beta\alpha$	V_{h}^{6}	nP2gg
pbmm	$p2/m2_1/m2/a$	40	$D_{2h}\overline{p}\,\mu\mu\alpha$	V_{h}^{3}	mP2mg
pmma	$p2/a2_1/m2/m$	41	$D_{2h}\overline{p}\,\alpha\mu\mu$	V_{h}^{9}	aP2mm
pbmn	$p2/n2/m2_1/a$	42	$D_{2h}\overline{p}\,\upsilon\mu\alpha$	V_{h}^{11}	nP2mg
pbaa	$p2/a2/b2_1/a$	43	$D_{2h}\overline{p}\,\alpha\beta\alpha$	V_{h}^{10}	aP2gg
pbam	$p2/m2_1/b2_1/a$	44	$D_{2h}\overline{p}\,\mu\beta\alpha$	V_{h}^{2}	mP2gg
pmab	$p2/a2_1/b2_1/m$	45	$D_{2h}\overline{p}\,\alpha\beta\mu$	V_{h}^{7}	aP2gm
pmmn	$p2/n2_1/m2_1/m$	46	$D_{2h}\overline{p}\,\upsilon\mu\mu$	V_{h}^{8}	nP2mm
cmmm	c2/m2/m2/m	47	$D_{2h}\overline{c}\,\mu\mu\mu$	V_{h}^{4}	mC2mm
cmma	c2/a2/m2/m	48	$D_{2h}\overline{c}\,\alpha\mu\mu$	V_{h}^{12}	aC2mm
p4	p4	58	$C_{4}\overline{p}$	C_{4}^{1}	1P4
$p\bar{4}$	$p\bar{4}$	57	$S_{4}\overline{p}$	S_{4}^{1}	$1P\bar{4}$
p4/m	p4/m	61	$C_{4h}\overline{p}\,\mu$	C_{4h}^{1}	mP4
p4/n	p4/n	62	$C_{4h}\overline{p}\,\upsilon$	C_{4h}^{2}	nP4

p422	p422	67	$D_4\overline{p}$11	D_4^1	1P422
p42$_1$2	p42$_1$2	68	$D_4\overline{p}$21	D_4^2	1P42$_1$2
p4mm	p4mm	59	$C_{4v}\overline{p}$μμ	C_{4v}^1	1P4mm
p4bm	p4bm	60	$C_{4v}\overline{p}$βμ	C_{4v}^2	1P4gm
p$\bar{4}$2m	p$\bar{4}$2m	63	$D_{2d}\overline{p}$μ1	V_d^1	1P$\bar{4}$2m
p$\bar{4}$2$_1$m	p$\bar{4}$2$_1$m	64	$D_{2d}\overline{p}$μ2	V_d^2	1P$\bar{4}$2$_1$m
p$\bar{4}$m2	p$\bar{4}$m2	65	$D_{2d}\overline{c}$μ1	V_d^3	1P$\bar{4}$m2
p$\bar{4}$b2	p$\bar{4}$b2	66	$D_{2d}\overline{c}$β1	V_d^4	1P$\bar{4}$g2
p4/mmm	p4/m2/m2/m	69	$D_{4h}\overline{p}$μμμ	D_{4h}^1	mP4mm
p4/nbm	p4/n2/b2/m	70	$D_{4h}\overline{p}$υβμ	D_{4h}^2	nP4gm
p4/mbm	p4/m2$_1$/b2/m	71	$D_{4h}\overline{p}$μβμ	D_{4h}^3	mP4gm
p4/nmm	p4/n2$_1$/m2/m	72	$D_{4h}\overline{p}$υμμ	D_{4h}^4	nP4mm
p3	p3	49	$C_3\overline{c}$	C_3^1	1P3
p$\bar{3}$	p$\bar{3}$	50	$S_6\overline{p}$	C_{3i}^1	1P$\bar{3}$
p312	p312	54	$D_3\overline{c}$1	D_3^1	1P312
p321	p321	53	$D_3\overline{h}$1	D_3^2	1P321
p3m1	p3m1	51	$C_{3v}\overline{c}$μ	C_{3v}^2	1P3m1
p31m	p31m	52	$C_{3v}\overline{h}$μ	C_{3v}^1	1P31m
p$\bar{3}$1m	p$\bar{3}$12/m	55	$D_{3d}\overline{c}$μ1	D_{3d}^2	1P$\bar{3}$1m
p$\bar{3}$m1	p$\bar{3}$2/m1	56	$D_{3d}\overline{h}$μ1	D_{3d}^1	1P$\bar{3}$m1
p6	p6	76	$C_6\overline{c}$	C_6^1	1P6
p$\bar{6}$	p$\bar{6}$	73	$C_{3h}\overline{c}$μ	C_{3h}^1	mP3
p6/m	p6/m	78	$C_{6h}\overline{c}$μ	C_{6h}^1	mP6
p622	p622	79	$D_6\overline{c}$11	D_6^1	1P622
p6mm	p6mm	77	$C_{6v}\overline{c}$μμ	C_{6v}^1	1P6mm
p$\bar{6}$m2	p$\bar{6}$m2	74	$D_{3h}\overline{c}$μμ	D_{3h}^1	mP3m2
p$\bar{6}$2m	p$\bar{6}$2m	75	$D_{3h}\overline{h}$μμ	D_{3h}^2	mP32m
p6/mmm	p6/m2/m2/m	80	$D_{6h}\overline{c}$μμμ	D_{6h}^1	mP6mm

14	15	16	17	18	19
(a/b)·1	1p1	p1	p1	p1	47
(a/b)·$\bar{1}$	1p$\bar{1}$	p2'	p2'	p2'	1
(a/b):2	1p112	p2	p2	p2	48
(a/b)·m	mp1	$p^{*}1$		p1'	64
(a/b)·$\tilde{b}$	bp1	$p_{b'}'1$	$p_{b}'1$	$p_{b}'1$	2
(a/b)·m:2	mp112	$p^{*}2$		p21'	65
(a/b)·$\tilde{b}$:2	bp112	$p_{b'}'2$	$p_{b}'2$	$p_{b}'2$	3
(a:b)·2	1p12	p1m'1	pm'	pm'	4
(a:b)·2_1	1p12_1	p1g'1	pg'	pg'	5
$(\frac{a+b}{2}/a:b)\cdot 2$	1c12	c1m'1	cm'	cm'	6
(a:b):m	1p1m	p11m	pm	pm	49
(a:b):$\tilde{a}$	1p1a	p11g	pg	pg	50
$(\frac{a+b}{2}/a:b):m$	1c1m	c11m	cm	cm	51
(a:b)·2:m	1p12/m	p2'm'm	pm'm	pmm'	14
(a:b)·2_1:m	1p12_1/m	p2'g'm	pg'm	pmg'	17
(a:b)·2·$\tilde{a}$	1p12_1/a	p2'g'g	pg'g	pgg'	18
(a:b)·2_1:$\tilde{a}$	p12/a	p2'm'g	pm'g	pm'g	16
$(\frac{a+b}{2}/a:b)\cdot 2:m$	1c12/m	c2'm'm	cm'm	cmm'	21
(a:b):2:2	1p222	p2m'm'	pm'm'	pm'm'	15
(a:b):2:2_1	1p$22_1$2	p2gm	pm'g'	pm'g'	20
(a:b)·2_1:2_1	1p$2_1 2_1$2	p2gg	pg'g'	pg'g'	19
$(\frac{a+b}{2}/a:b):2:2$	1c222	c2m'm'	cm'm'	cm'm'	22
(a:b):2·m	1pmm2	p2mm	pmm	pmm2	52
(a:b):2·$\tilde{b}$	1pma2	p2mg	pmg	pmg2	53
(a:b):$\tilde{a}$:$\tilde{b}$	1pba2	p2gg	pgg	pgg2	54

$(\frac{a+b}{2}/a{:}b){:}m\cdot 2$	1cmm2	c2mm	cmm	cmm2	55
$(a{:}b)\cdot m\cdot 2$	mpm2	$p^{*}1m1$		pm1'	66
$(a{:}b){:}m\cdot 2_1$	$bpm2_1$	$p_{b'}'1m1$	$p_a'1m$	$p_b'm$	7
$(a{:}b)\cdot m\cdot 2_1$	$mpb2_1$	$p^{*}1g1$		pg1'	67
$(a{:}b)\cdot\tilde{a}\cdot 2$	bpb2	$p_{b'}'1m'1$	$p_a'1g$	$p_b'g$	8
$(a{:}b)\cdot\tilde{b}\cdot 2$	apm2	$p_{a'}'1m1$	$p_b'1m$	$p_b'1m$	9
$(a{:}b)\cdot ab\cdot 2_1$	$npm2_1$	c'1m1	$p_c'1m$	$p_c'm$	11
$(a{:}b)\cdot\tilde{b}{:}\tilde{a}$	$apb2_1$	$p_{a'}'1g1$	$p_b'1g$	$p_b'1g$	10
$(a{:}b)\cdot ab\cdot 2$	npb2	c'1m'1	$p_c'1m'$	$p_c'g$	12
$(\frac{a+b}{2}/a{:}b)\cdot m\cdot 2$	mcm2	$c^{*}1m1$		cm1'	68
$(\frac{a+b}{2}/a{:}b)\cdot\tilde{b}\cdot 2$	acm2	$p_{a'b'}'1m1$	c'1m	c'm	13
$(a{:}b)\cdot m{:}2\cdot m$	mp2/m2/m2	$p^{*}2mm$		pmm21'	69
$(a{:}b)\cdot\tilde{a}{:}2\cdot\tilde{a}$	ip2/m2/a2	$p_{a'}'2mg$	$p_a'mg$	$p_b'gm$	25
$(a{:}b)\cdot ab{:}2\cdot a$	np2/b2/a2	c'2m'm'	$p_c'm'm'$	$p_c'gg$	29
$(a{:}b)\cdot m{:}2\cdot\tilde{b}$	$np2_1/m2/a2$	$p^{*}2mg$		pmg21'	70
$(a{:}b)\cdot\tilde{a}{:}2\cdot m$	$ap2_1/m2/m2$	$p_{a'}'2mm$	$p_b'mm$	$p_b'mm$	23
$(a{:}b)\cdot ab{:}2\cdot b$	$np2/m2_1/a2$	c'2mm'	$p_c'm'm$	$p_c'mg$	28
$(a{:}b)\cdot\tilde{a}\cdot 2{:}\tilde{b}$	$ap2/b2_1/a2$	$p_{a'}'2gg$	$p_b'gg$	$p_b'gg$	26
$(a{:}b)\cdot m{:}\tilde{a}{:}\tilde{b}$	$np2_1/b2_1/a2$	$p^{*}2gg$		pgg21'	71
$(a{:}b)\cdot\tilde{b}{:}2\cdot\tilde{a}$	$ap2_1/b2_1/m2$	$p_{a'}'2gm$	$p_b'mg$	$p_b'mg$	24
$(a{:}b)\cdot ab{:}2\cdot m$	$np2_1/m2_1/m2$	c'2mm	$p_c'mm$	$p_c'mm$	27
$(\frac{a+b}{2}/a{:}b)\cdot m{:}2\cdot m$	mc2/m2/m2	$c^{*}2mm$		cmm21'	72
$(\frac{a+b}{2}/a{:}b)\cdot\tilde{a}{:}2\cdot m$	ac2/m2/m2	$p_{a'b'}'2mm$	c'mm	c'mm	30
(a:a):4	1p4	p4	p4	p4	56
$(a{:}a){:}\bar{4}$	$1p\bar{4}$	p4'	p4'	p4'	31
(a:a):4:m	mp4	$p^{*}4$		p41'	73
(a:a):4:ab	np4	c'4	p'4	$p_c'4$	32

(a:a):4:2	1p422	p4m'm'	p4m'm'	p4m'm'	35
(a:a):4:2_1	1p4$2_1$2	p4g'm'	p4g'm'	p4g'm'	38
(a:a):4·m	1p4mm	p4mm	p4mm	p4mm	57
(a:a):4$\odot$b	1p4bm	p4gm	p4gm	p4gm	58
(a:a):$\bar{4}$:2	1p$\bar{4}$2m	p4'm'm	p4'm'm	p4'm'm	34
(a:a):$\bar{4}\odot 2_1$	1p$\bar{4}2_1$m	p4'g'm	p4'g'm	p4'g'm	37
(a:a):$\bar{4}$·m	1p$\bar{4}$m2	p4'mm'	p4'mm'	p4'mm'	33
(a:a):$\bar{4}\odot\tilde{b}$	1p$\bar{4}$b2	p4'gm'	p4'gm'	p4'gm'	36
(a:a)·m:4·m	mp42/m2/m	p^*4mm		p4mm1'	74
(a:a):ab:4$\odot$b	np42/b2/m	c'4m'm	p'4gm	p_c'4gm	40
(a:a)·m:4$\odot$b	mp4$2_1$/b2/m	p^*4gm		p4gm1'	75
(a:a)·ab:4·m	np4$2_1$/m2/m	c'4mm	p'4mm	p_c'4mm	39
(a/a):3	1p3	p3	p3	p3	59
(a/a):$\bar{3}$	1p$\bar{3}$	p6'	p6'	p6'	43
(a/a):2:3	1p312	p3m'1	p3m'1	p3m'	41
(a/a)·2:3	1p321	p31m'	p31m'	p31m'	42
(a/a):m·3	1p3m1	p3m1	p3m1	p3m	60
(a/a)·m·3	1p31m	p31m	p31m	p31m	61
(a/a)·m·$\bar{6}$	1p$\bar{3}$12/m	p6'm'm	p6'm'm	p6'm'm	44
(a/a):m·$\bar{6}$	1p$\bar{3}$2/m1	p6'mm'	p6'mm'	p6'mm'	45
(a/a):6	1p6	p6	p6	p6	62
(a/a):3:m	mp3	p^*3		p3'	76
(a/a)·m:6	mp6	p^*6		p61'	79
(a/a)·2:6	1p622	p6m'm'	p6m'm'	p6m'm'	46
(a/a):m·6	1p6mm	p6mm	p6mm	p6mm	63
(a/a):m·3:m	mp3m2	p^*3m1		p3'm	77
(a/a)·m:3·m	mp32m	p^*31m		p3'1m	78
(a/a)·m:6·m	mp6mm	p^*6mm		p6mm1'	80

20	21	22	23	24	25
		p1			
p2'	p2$^-$	p2'	p2[2]$_1$	2'11	p2/p1
		p2			
		p11'			
pt'	pt$^-$	p$_{2b}$1	p1[2]	b11	p1/p1
		p21'			
p2t'	p2t$^-$	p$_{2b}$2	p2[2]$_2$	2/b11	p2/p2
pm'	pm$^-$	pm'	pm[2]$_4$	12'1	pm/p1
pg'	pg$^-$	pg'	pg[2]$_1$	112$_1$'	pg/p1
cm'	cm$^-$	cm'	cm[2]$_1$	c112'	cm/p1
		pm			
		pg			
		cm			
pmm'	pmm$^-$	pm'm	pmm[2]$_2$	2'2'2	pmm/pm
pmg'	pmg$^-$	pmg'	pmg[2]$_4$	2'2$_1$'2	pmg/pm
pgg'	pgg$^-$	pgg'	pgg[2]$_1$	2'2$_1$'2$_1$	pgg/pg
pm'g	pm$^-$g	pm'g	pmg[2]$_2$	2'2$_1$2'	pmg/pg
cmm'	cmm$^-$	cmm'	cmm[2]$_2$	c2'22'	cmm/cm
pm'm'	pm$^-$m$^-$	pm'm'	pmm[2]$_5$	22'2'	pmm/p2
pm'g'	pm$^-$g$^-$	pm'g'	pmg[2]$_5$	22'2$_1$'	pmg/p2
pg'g'	pg$^-$g$^-$	pg'g'	pgg[2]$_2$	22$_1$'2$_1$'	pgg/p2
cm'm'	cm$^-$m$^-$	cm'm'	cmm[2]$_4$	c22'2'	cmm/p2
		pmm			
		pmg			
		pgg			

		cmm			
		pm1'			
pm+t'	$pm+t^-$	$p_{2b}m$	$pm[2]_3$	b12	pm/pm(m)
		pg1'			
pg+t'	$pg+t^-$	$p_{2b}m'$	$pm[2]_1$	$b12_1$	pm/pg
pm+m'	$pm+m^-$	$p_{2a}m$	$pm[2]_5$	b'1m	pm/pm(m')
pm+g'	$pm+g^-$	c_pm	$cm[2]_3$	n12	cm/pm
pg+g'	$pg+g^-$	$p_{2a}g$	$pg[2]_2$	$b2_11$	pg/pg
pg+m'	$pg+m^-$	c_pm'	$cm[2]_2$	$n12_1$	cm/pg
		cm1'			
cm+m'	$cm+m^-$	p_cm	$pm[2]_2$	ca12	pm/cm
		pmm1'			
pg,m+m'	$pg,m+m^-$	$p_{2a}mm'$	$pmm[2]_4$	$a2_12$	pmm/pmg
pg+m',g+m'	$pg+m^-,g+m^-$	$c_pm'm'$	$cmm[2]_1$	$n2_12_1$	cmm/pgg
		pmg1'			
pm,m+m'	$pm,m+m^-$	$p_{2a}mm$	$pmm[2]_1$	a22	pmm/pmm
pm+g',g+m'	$pm+g^-,g+m^-$	c_pmm'	$cmm[2]_3$	$n22_1$	cmm/pmg
pg,g+g'	$pg,g+g^-$	$p_{2b}m'g$	$pmg[2]_3$	$a2_12_1$	pmg/pgg
		pgg1'			
pm,g+g'	$pm,g+g^-$	$p_{2b}mg$	$pmg[2]_1$	$b2_12$	pmg/pmg
pm+g',m+g'	$pm+g^-,m+g^-$	c_pmm	$cmm[2]_5$	n22	cmm/pmm
		cmm1'			
cm+m',m+m'	$cm+m^-,m+m^-$	p_cmm	$pmm[2]_3$	ca22	pmm/cmm
		p4			
p4'	$p4^-$	p4'	$p4[2]_2$	4'11	p4/p2
		p41'			
p4t'	$p4t^-$	p_p4	$p4[2]_1$	4/n11	p4/p4

p4m'm'	p4m$^-$m$^-$	p4m'	pm4$[2]_2$	42'2'	p4m/p4
p4g'm'	p4g$^-$m$^-$	p4g'	p4g$[2]_1$	4$2_1$'2'	p4g/p4
		p4m			
		p4g			
p4'm'm	p4$^-$m$^-$m	p4'm'	p4m$[2]_3$	4'2'2	p4m/cmm
p4'g'm	p4$^-$g$^-$m	p4'g'	p4g$[2]_2$	4'2_1'2	p4g/cmm
p4'mm'	p4$^-$mm$^-$	p4'm	p4m$[2]_4$	4'22'	p4m/pmm
p4'gm'	p4$^-$gm$^-$	p4'g	p4g$[2]_3$	4'$2_1$2'	p4g/pgg
		p4m1'			
p4g+m',m+m'	p4g+m$^-$,m+m$^-$	p_p4m'	p4m$[2]_1$	4/n$2_1$2	p4m/p4g
		p4g1'			
p4m+g',m+m'	p4m+g$^-$,m+m$^-$	p_p4m	p4m$[2]_5$	4/n22	p4m/p4m
		p3			
p6'	p6$^-$	p6'	p6[2]	6'	p6/p3
p3m'1	p3m$^-$1	p3m'1	p3m1[2]	312'	p3m1/p3
p31m'	p31m$^-$	p31m'	p31m[2]	32'1	p31m/p3
		p3m1			
		p31m			
p6'm'm	p6$^-$m$^-$m	p6'm'	p6m$[2]_1$	6'22'	p6m/p31m
p6'mm'	p6$^-$mm$^-$	p6'm	p6m$[2]_2$	6'2'2	p6m/p3m1
		p6			
		p31'			
		p61'			
p6m'm'	p6m$^-$m$^-$	p6m'	p6m$[2]_3$	62'2'	p6m/p6
		p6m			
		p3m11'			
		p31m1'			
		p6m1'			

2. Rod Group Symbols

	1	2	3	4	5
Triclinic	1	*p*1	1	P(11)1	1
	2	$p\bar{1}$	2	$P(\bar{1}\bar{1})\bar{1}$	7
Monoclinic/ Inclined	3	*p*211	6	P(12)1	2
	4	*p*m11	3	P(1m)1	22
	5	*p*c11	5	P(1c)1	24
	6	*p*2/m11	9	P(12/m)1	25
	7	*p*2/c11	12	P(12/c)1	28
Monoclinic/ Orthogonal	8	*p*112	7	P(11)2	3
	9	$p112_1$	8	$P(11)2_1$	8
	10	*p*11m	4	P(11)m	23
	11	*p*112/m	10	P(11)2/m	26
	12	$p112_1/m$	11	$P(11)2_1/m$	27
Orthorhombic	13	*p*222	18	P(22)2	61
	14	$p222_1$	19	$P(22)2_1$	62
	15	*p*mm2	13	P(mm)2	34
	16	*p*cc2	16	P(cc)2	35
	17	$pmc2_1$	15	$P(mc)2_1$	36
	18	*p*22m	14	P(2m)m	33
	19	*p*2cm	17	P(2c)m	37
	20	*p*mmm	20	P(2/m2/m)2/m	46
	21	*p*ccm	21	P(2/c2/c)2/m	47
	22	*p*mcm	22	$P(2/m2/c)2_1/m$	48
Tetragonal	23	*p*4	26	P4(11)	5
	24	$p4_1$	27	$P4_1(11)$	11

		25	$p4_2$	28	$P4_2(11)$	12
		26	$p4_3$	29	$P4_3(11)$	13
		27	$p\bar{4}$	23	$P\bar{4}(11)$	20
		28	$p4/m$	30	$P4/m(11)$	29
		29	$p4_2/m$	31	$P4_2/m(11)$	30
		30	$p422$	35	$P4(22)$	66
		31	$p4_122$	36	$P4_1(22)$	67
		32	$p4_222$	37	$P4_2(22)$	68
		33	$p4_322$	38	$P4_3(22)$	69
		34	$p4mm$	32	$P4(mm)$	40
		35	$p4_2cm$	33	$P4_2(cm)$	42
		36	$p4cc$	34	$P4(cc)$	41
		37	$p\bar{4}2m$	24	$P\bar{4}(2m)$	49
		38	$p\bar{4}2c$	25	$P\bar{4}(2c)$	50
	53	39	$p4/mmm$	39	$P4/m(2/m2/m)$	
	54	40	$p4/mmc$	40	$P4/m(2/c2/c)$	
	55	41	$p4_2/mmc$	41	$P4_2/m(2/m2/c)$	
Trigonal		42	$p3$	42	$P3(11)$	4
		43	$p3_1$	43	$P3_1(11)$	9
		44	$p3_2$	44	$P3_2(11)$	10
		45	$p\bar{3}$	45	$P\bar{3}(11)$	19
		46	$p312$	48	$P3(21)$	63
		47	$p3_112$	49	$P3_1(21)$	64
		48	$p3_212$	50	$P3_2(21)$	65
		49	$p3m1$	46	$P3(m1)$	38
		50	$p3c1$	47	$P3(c1)$	39
		51	$p\bar{3}1m$	51	$P\bar{3}(m1)$	59

		52	$p\bar{3}1c$	52	$P\bar{3}(c1)$	60
Hexangonal		53	$p6$	56	$P6(11)$	6
		54	$p6_1$	57	$P6_1(11)$	14
		55	$p6_2$	59	$P6_2(11)$	15
		56	$p6_3$	61	$P6_3(11)$	16
		57	$p6_4$	60	$P6_4(11)$	17
		58	$p6_5$	58	$P6_5(11)$	18
		59	$p\bar{6}$	53	$P\bar{6}(11)$	21
		60	$p6/m$	62	$P6/m(11)$	31
		61	$p6_3/m$	63	$P6_3/m(11)$	32
		62	$p622$	67	$P6(22)$	70
		63	$p6_122$	68	$P6_1(22)$	71
		64	$p6_222$	70	$P6_2(22)$	72
		65	$p6_322$	72	$P6_3(22)$	73
		66	$p6_422$	71	$P6_4(22)$	74
		67	$p6_522$	69	$P6_5(22)$	75
		68	$p6mm$	64	$P6(mm)$	43
		69	$p6cc$	65	$P6(cc)$	44
		70	$p6_3mc$	66	$P6_3(cm)$	45
		71	$p\bar{6}m2$	54	$P\bar{6}(m2)$	51
		72	$p\bar{6}c2$	55	$P\bar{6}(c2)$	52
	56	73	$p6/mmm$	73	$P6/m(2/m2/m)$	
	57	74	$p6/mcc$	74	$P6/m(2/c2/c)$	
	58	75	$p6_3/mmc$	75	$P6_3/m(2/c2/m)$	

6	7	8	9
(a)·1	p1	r1	1P1
(a)·$\bar{1}$	p$\bar{1}$	r$\bar{1}$	1P$\bar{1}$
(a):2	p112	r112	1P2
(a)·m	p11m	r1m1	mP1
(a)·$\tilde{a}$	p11a	r1c1	gP1
(a):2:m	p112/m	r12/m1	mP2
(a):2:$\tilde{a}$	p112/a	r12/c1	gP2
(a)·2	p211	r211	2P1
(a)·2_1	p2_1	r2_1	2_1P1
(a):m	pm11	rm11	1Pm
(a)·2:m	p2/m11	r2/m11	2Pm
(a)·2_1:m	p2_1/m11	r2_1/m11	2_1Pm
(a)·2:2	p222	r222	2P22
(a)·2_1:2	p$2_1$22	r$2_1$22	2_1P22
(a)·2·m	p2mm	r2mm	2mmP1
(a)·2·$\tilde{a}$	p2aa	r2cc	2ggP1
(a)·2_1·m	p2_1ma	r2_1mc	2_1mgP1
(a):2·m	pmma	rmm2	mPm2
(a):2·$\tilde{a}$	pma2	rmc2	gPm2
(a)·m·2:m	pmmm	r2/m2/m2/m	mmPm
(a)·$\tilde{a}$·2:m	pmaa	r2/m2/c2/c	ggPm
(a)·m·2_1:m	pmma	r2_1/m2/m2/c	mgPm
(a)·4	p4	r4	4P1
(a)·4_1	p4_1	r4_1	4_1P1

(a)·4_2	p4_2	r4_2	4_2P1
(a)·4_3	p4_3	r4_3	4_3P1
(a)·$\tilde{4}$	p$\overline{4}$	r$\overline{4}$	1P$\overline{4}$
(a)·4:m	p4/m	r4/m	4Pm
(a)·4_2:m	p4_2/m	r4_2/m	4_2Pm
(a)·4:2	p422	r422	4P22
(a)·4_1:2	p$4_1$22	r$4_1$22	4_1P22
(a)·4_2:2	p$4_2$22	r$4_2$22	4_2P22
(a)·4_3:2	p$4_3$22	r$4_3$22	4_3P22
(a)·4·m	p4mm	r4mm	4mmP1
(a)·4_2·m	p4_2ma	r4_2mc	4_2mgP1
(a)·4·$\tilde{a}$	p4aa	r4cc	4ggP1
(a)·$\tilde{4}$·m	p$\bar{4}$2m	r$\bar{4}$m2	mP$\bar{4}$2
(a)·$\tilde{4}$·$\tilde{a}$	p$\bar{4}$2a	r$\bar{4}$c2	gP$\bar{4}$2
(a)·m·4:m	p4/mmm	r4/m2/m2/m	4mmPm
(a)·$\tilde{a}$·4:m	p4/maa	r4/m2/c2/c	4ggPm
(a)·m·4_2:m	p4_2/mma	r4_2/m2/m2/c	4_2mgPm
(a)·3	p3	r3	3P1
(a)·3_1	p3_1	r3_1	3_1P1
(a)·3_2	p3_2	r3_2	3_2P1
(a)·$\tilde{6}$	p$\bar{3}$	r$\bar{3}$	3P$\bar{1}$
(a)·3:2	p32	r32	3P2
(a)·3_1:2	p$3_1$2	r$3_1$2	3_1P2
(a)·3_2:2	p$3_2$2	r$3_2$2	3_2P2
(a)·3·m	p3m	r3m	3mP1
(a)·3·$\tilde{a}$	p3a	r3c	3gP1
(a)·$\tilde{6}$·m	p$\bar{3}$m	r$\bar{3}$2/m	3mP$\bar{1}$2

(a)·$\tilde{6}$·$\tilde{a}$	p$\bar{3}$a	r$\bar{3}$2/c	3gP$\bar{1}$2
(a)·6	p6	r6	6P1
(a)·6_1	p6_1	r6_1	6_1P1
(a)·6_2	p6_2	r6_2	6_2P1
(a)·6_3	p6_3	r6_3	6_3P1
(a)·6_4	p6_4	r6_4	6_4P1
(a)·6_5	p6_5	r6_5	6_5P1
(a)·3:m	p$\bar{6}$	r$\bar{6}$	3Pm
(a)·6:m	p6/m	r6/m	6Pm
(a)·6_3:m	p6_3/m	r6_3/m	6_3Pm
(a)·6:2	p622	r622	6P22
(a)·6_1:2	p$6_1$22	r$6_1$22	6_1P22
(a)·6_2:2	p$6_2$22	r$6_2$22	6_2P22
(a)·6_3:2	p$6_3$22	r$6_3$22	6_3P22
(a)·6_4:2	p$6_4$22	r$6_4$22	6_4P22
(a)·6_5:2	p$6_5$22	r$6_5$22	6_5P22
(a)·6·m	p6mm	r6mm	6mmP1
(a)·6·$\tilde{a}$	p6aa	r6cc	6ggP1
(a)·6_3·m	p6_3ma	r6_3mc	6_3mgP1
(a)·m·3:m	p$\bar{6}$m2	r$\bar{6}$m2	3mPm2
(a)·$\tilde{a}$·3:m	p$\bar{6}$a2	r$\bar{6}$c2	3gPm2
(a)·m·6:m	p6/mmm	r6/m2/m2/m	6mmPm
(a)·$\tilde{a}$·6:m	p6/maa	r6/m2/c2/c	6ggPm
(a)·m·6_3:m	p6_3/mma	r6_3/m2/m2/c	6_3mgPm

3. Frieze Group Symbols

	1	2	3	4	5
Oblique	1	*p*1	r1	r1	r111
	2	*p*211	$r\bar{1}'$	r112	r112
Rectangular	3	*p*1m1	$r\bar{1}$	r1m	rm11
	4	*p*11m	r11'	rm	r1m1
	5	*p*11g	$r_2 1$	rg	r1c1
	6	*p*2mm	$r\bar{1}1'$	rmm2	rmm2
	7	*p*2mg	$r_2\bar{1}$	rgm2	rmc2

6	7	8	9	10	11
(a)	t	1	p[1](1)1	r1	*p*1
(a):2	t:2	5	p[2](1)1	r2	*p*112
(a):m	t:m	3	p[1](1)m	r1m	*p*m11
(a)·m	t·m	2	p[1](m)1	r11m	*p*1m1
(a)·$\tilde{a}$	t·a	4	p[1](c)1	r11g	*p*1a1
(a):2·m	t:2·m	6	p[2](m)m	r2mm	*p*mm2
(a):2·$\tilde{a}$	t:2·a	7	p[2](c)m	r2mg	*p*ma2